Increment-and-Freeze

Every Cache, Everywhere, All of the Time

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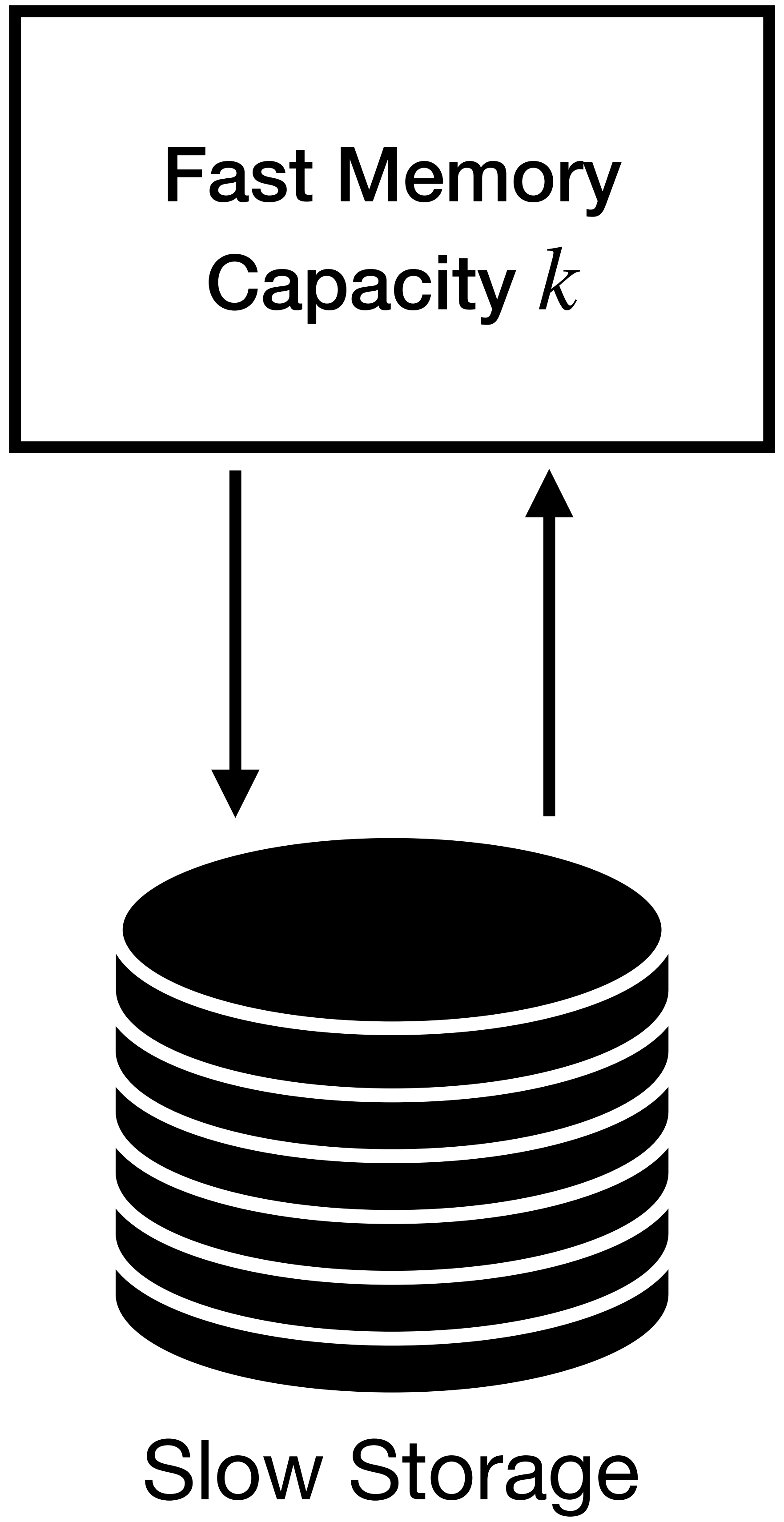
The Paging Problem

Foundation

- Stream of **page** requests, e.g. ABACB



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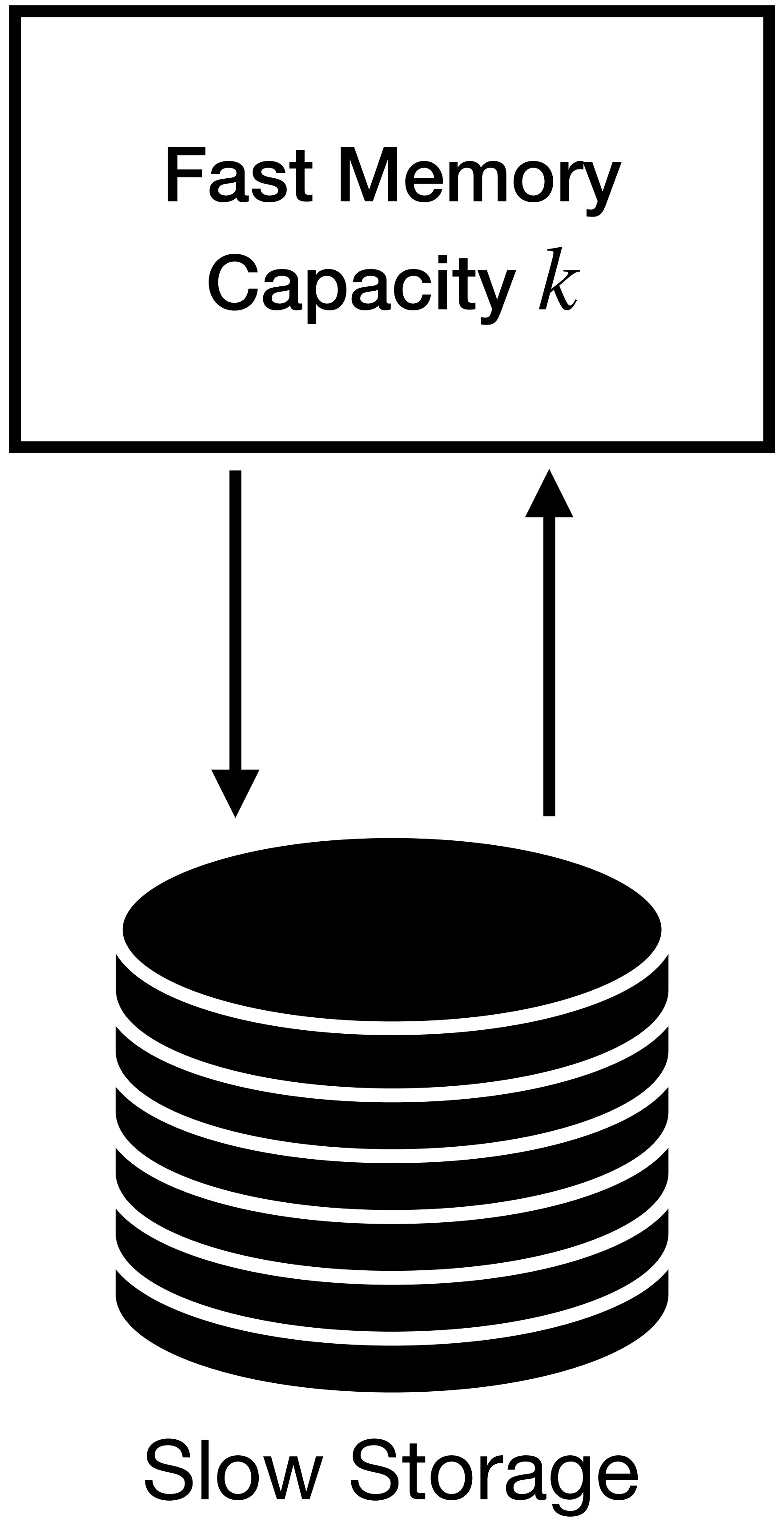
  - Fast **Hit** if page already cached, slow **miss** if not



The Paging Problem

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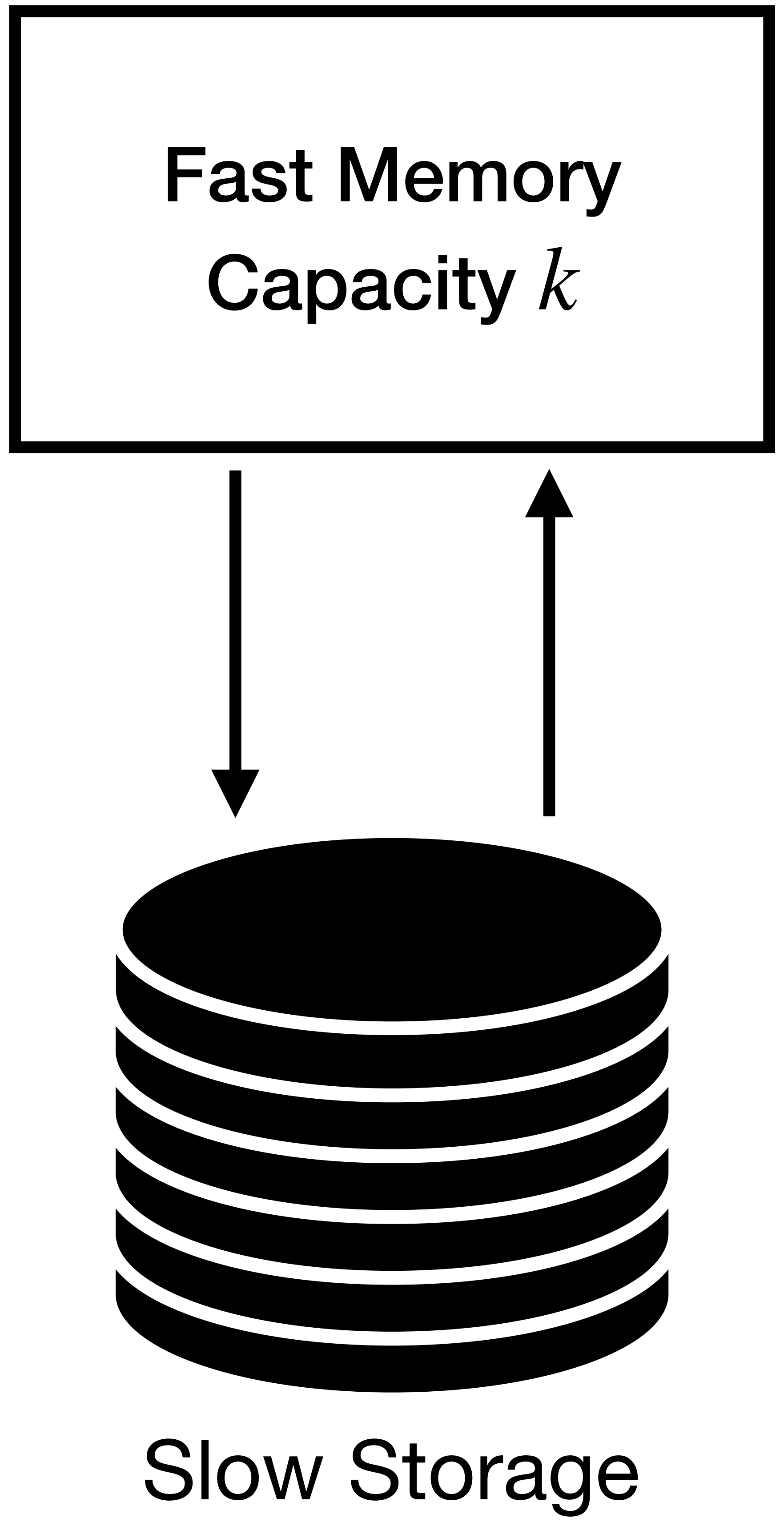
- Stream of **page** requests, e.g. ABACB
- Pages held within slow storage and must be cached in fast memory to be served
    - Fast **Hit** if page already cached, slow **miss** if not
- Algorithms for the paging problem make eviction decisions.
  Evicting the least recently used page is known solution



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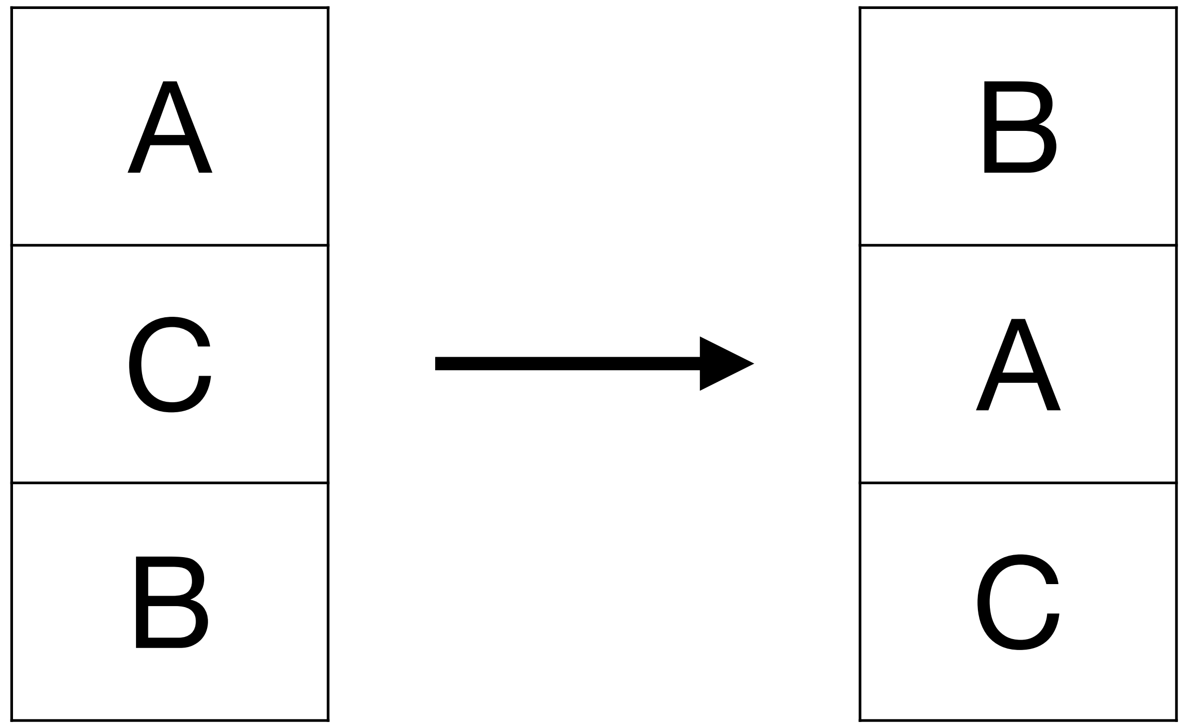

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Hit!

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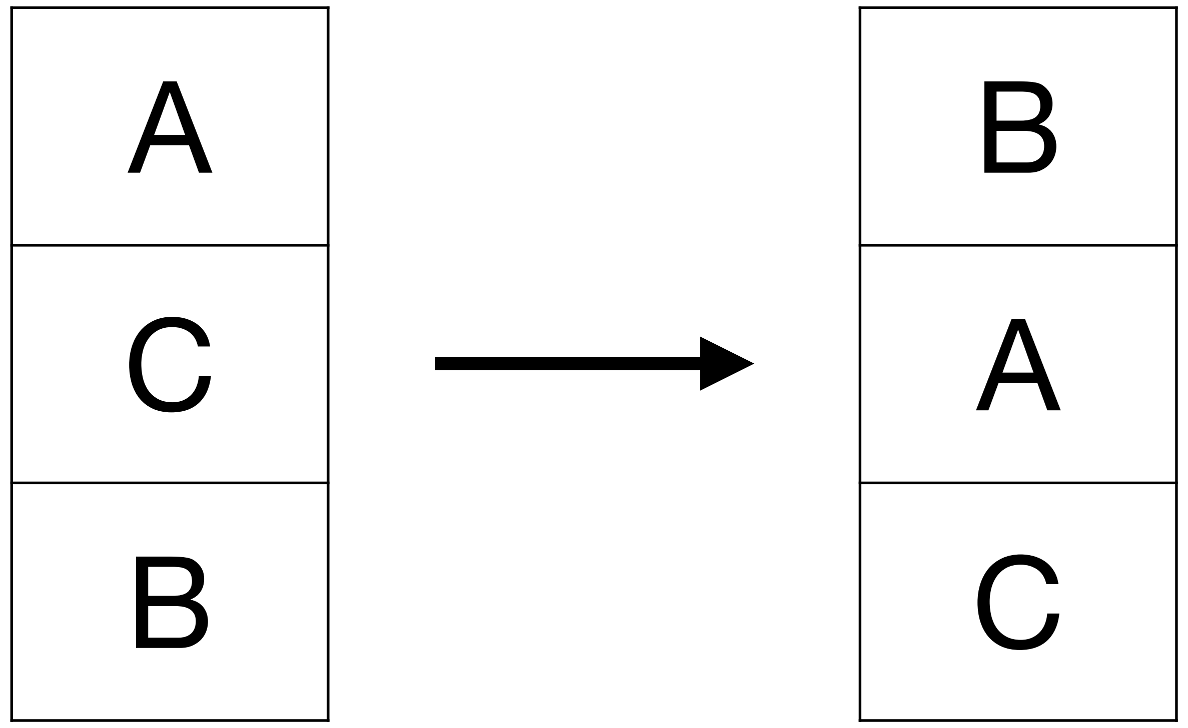

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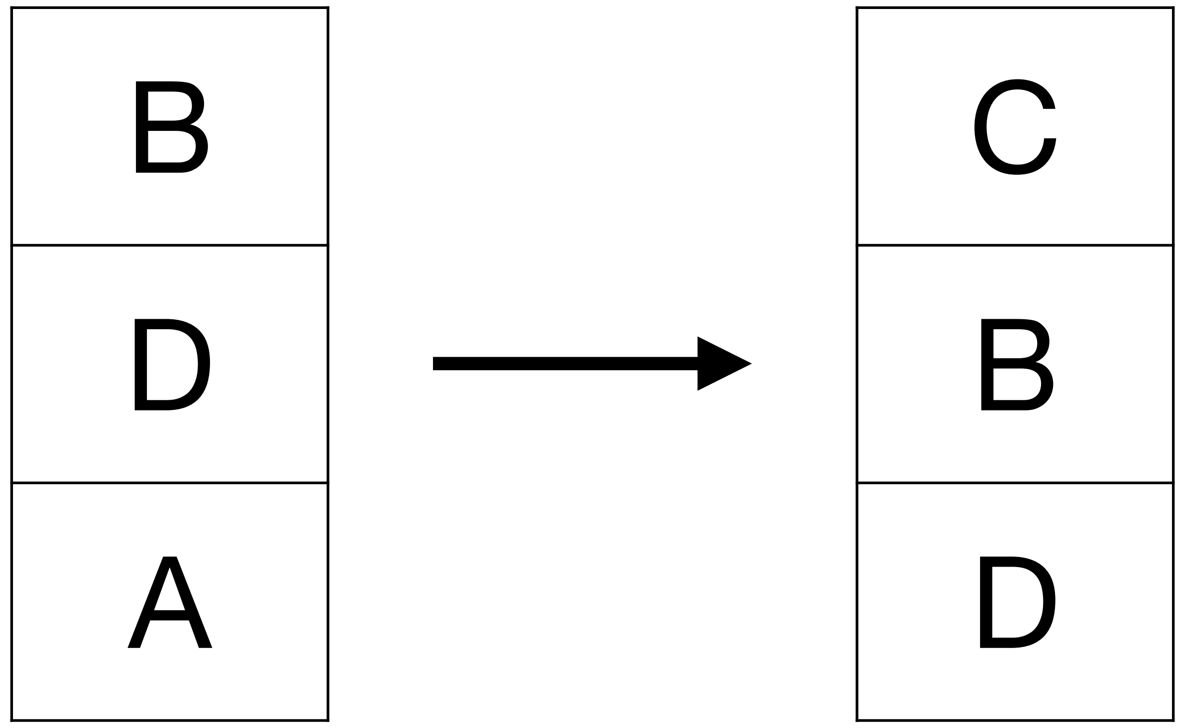


ABDBC
Miss!

LRU Hit-rate Curves

Simulating Caches with LRU-Hit Rate Curves

- LRU hit-rate curves give the hit rate of every cache size for a sequence of page requests

- Sequence of page requests generated by execution of some program

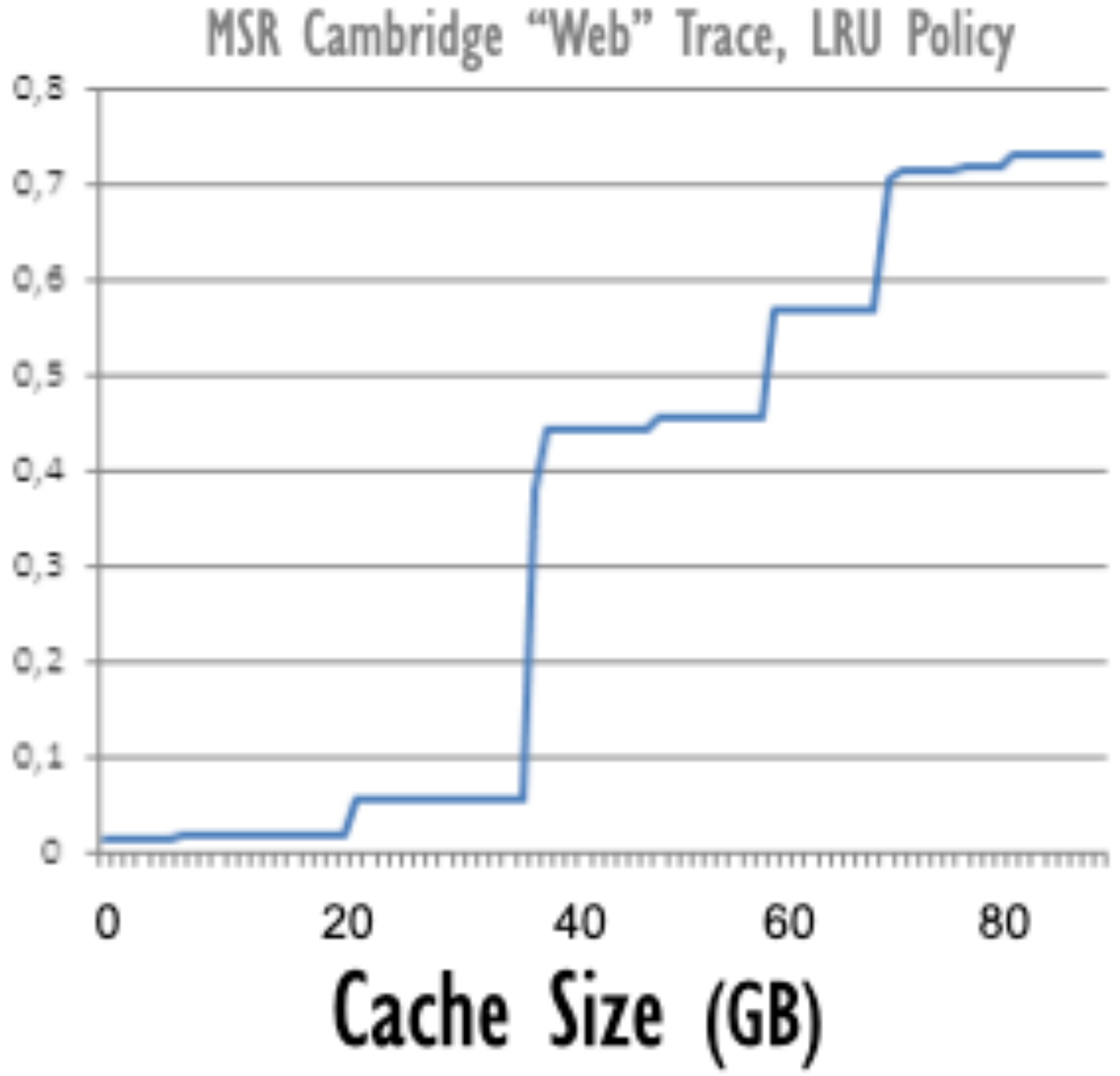
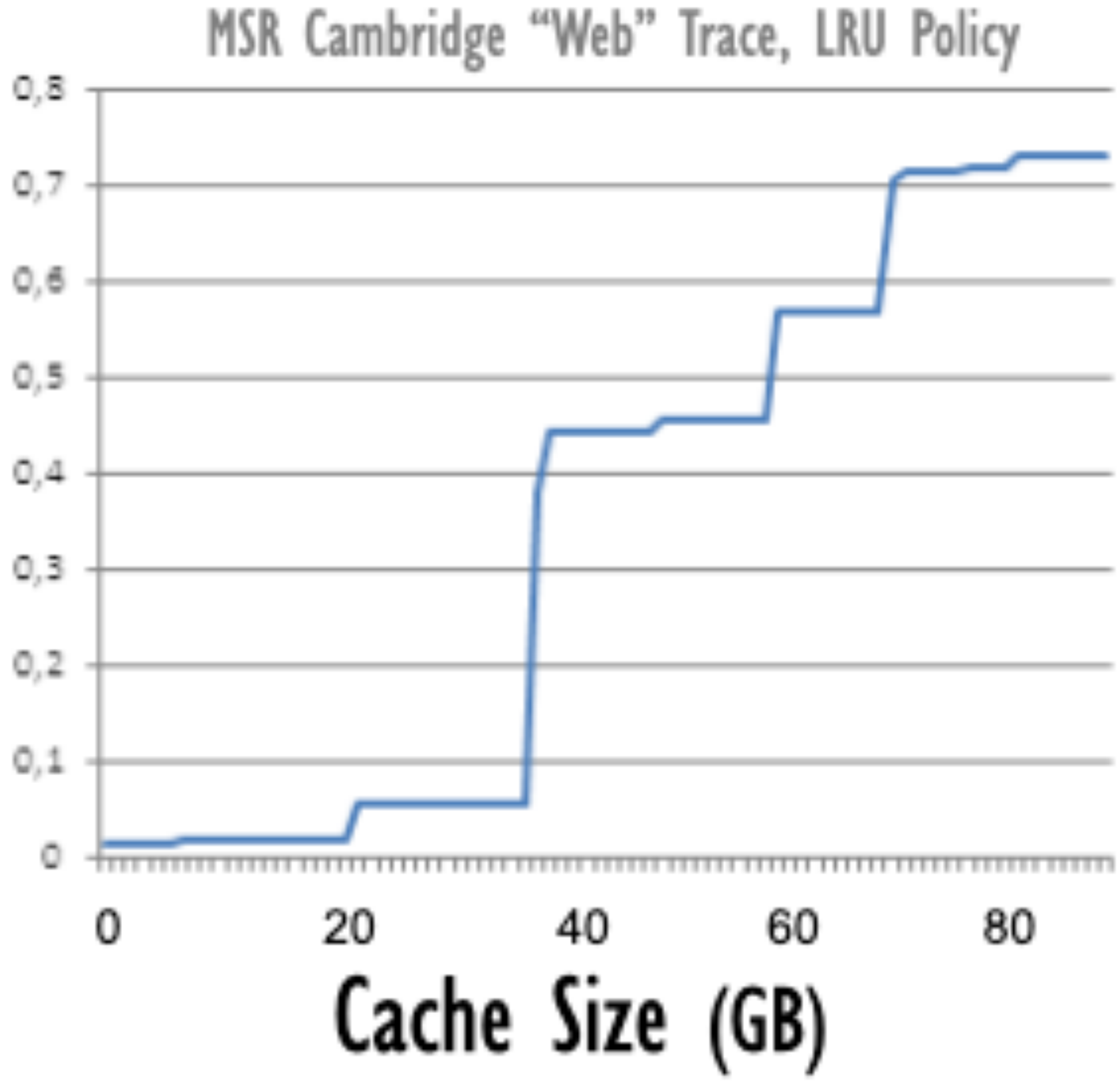


Image: [Druidi et al. 2015]

Got Cache Questions?

LRU hit-rate curves answer them

- The bigger the cache, the more expensive it is
- Misses are also expensive: user latency, server load



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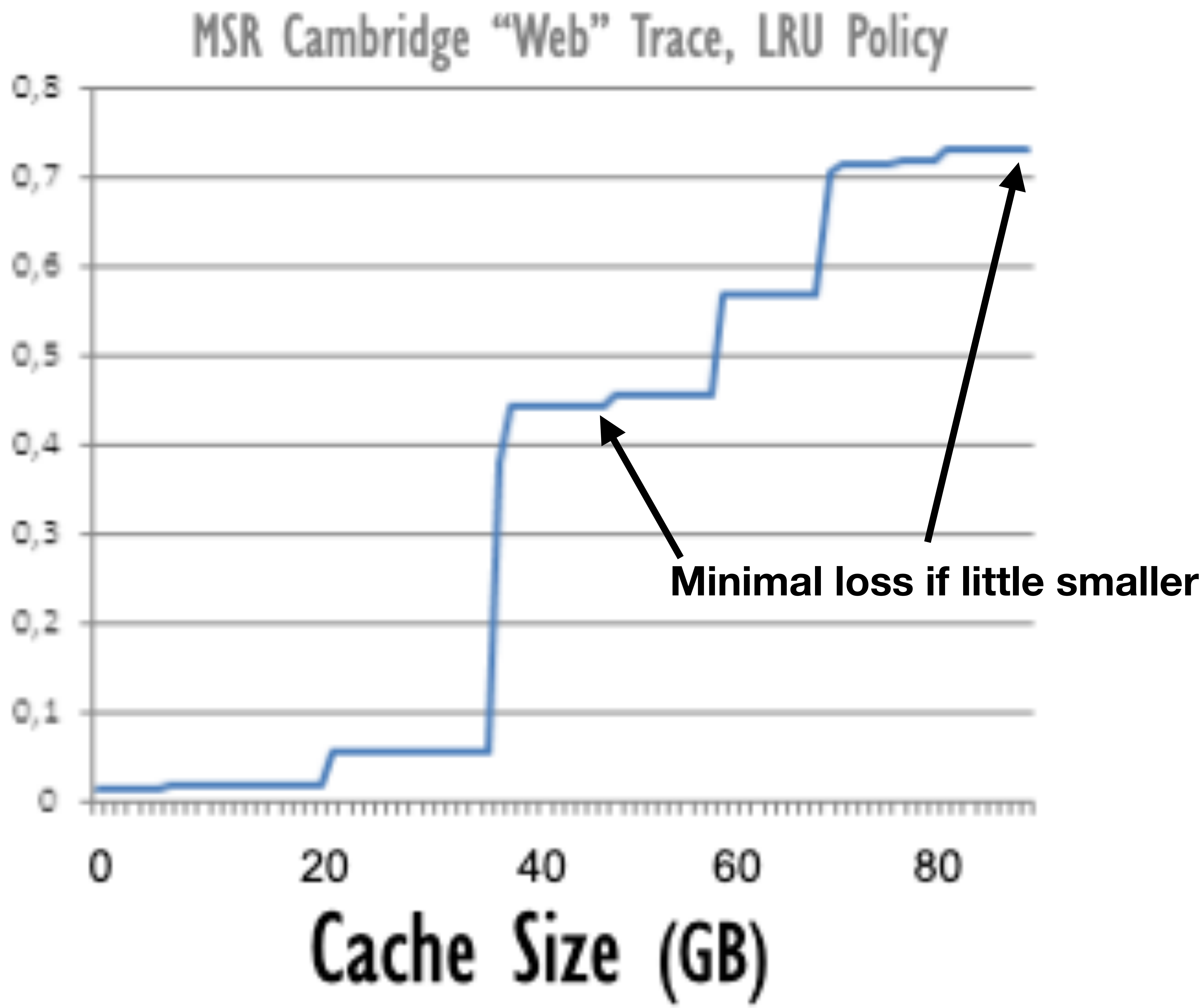

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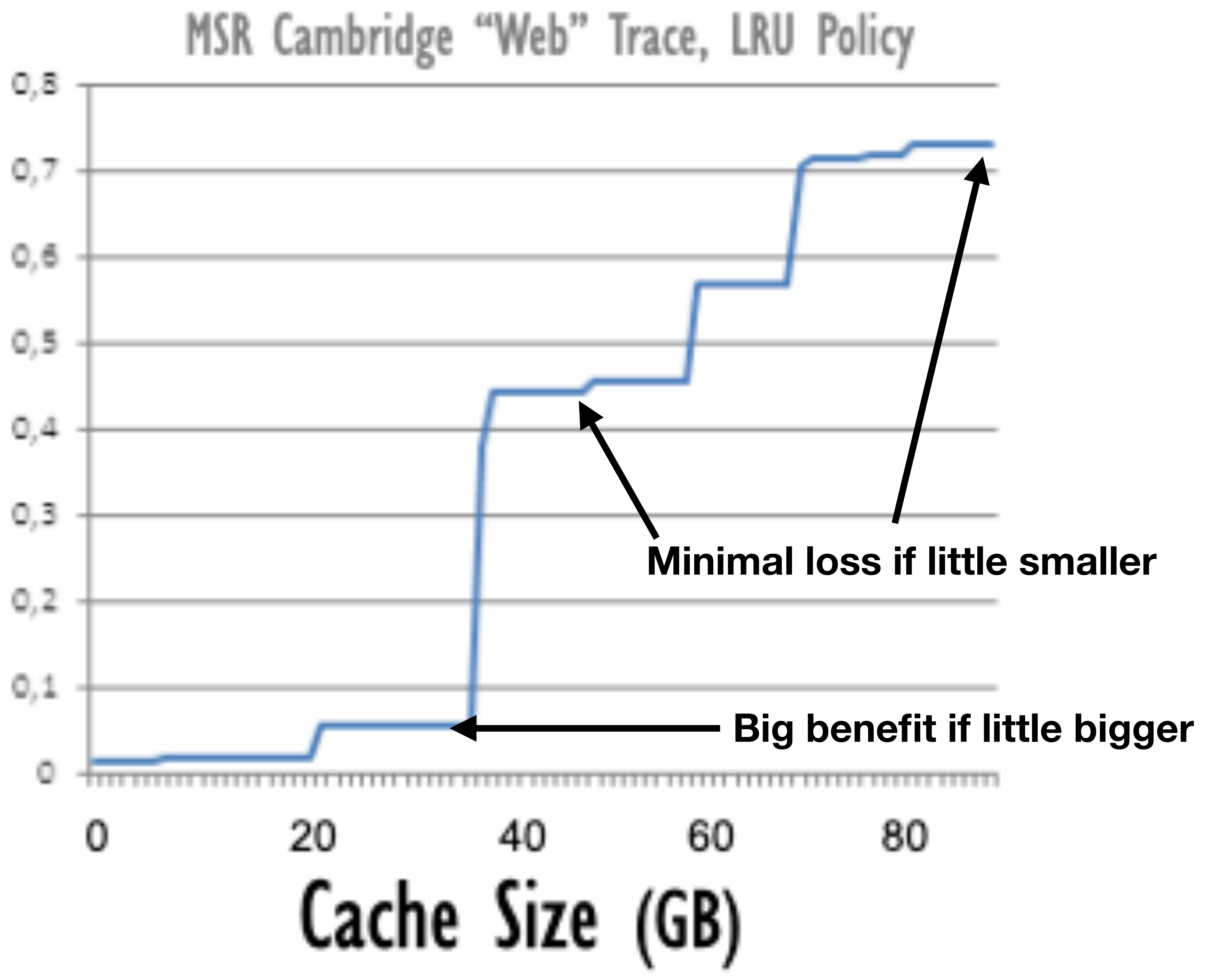
- Reduce cost by shrinking cache size?
- Improve hit rate via small increase?



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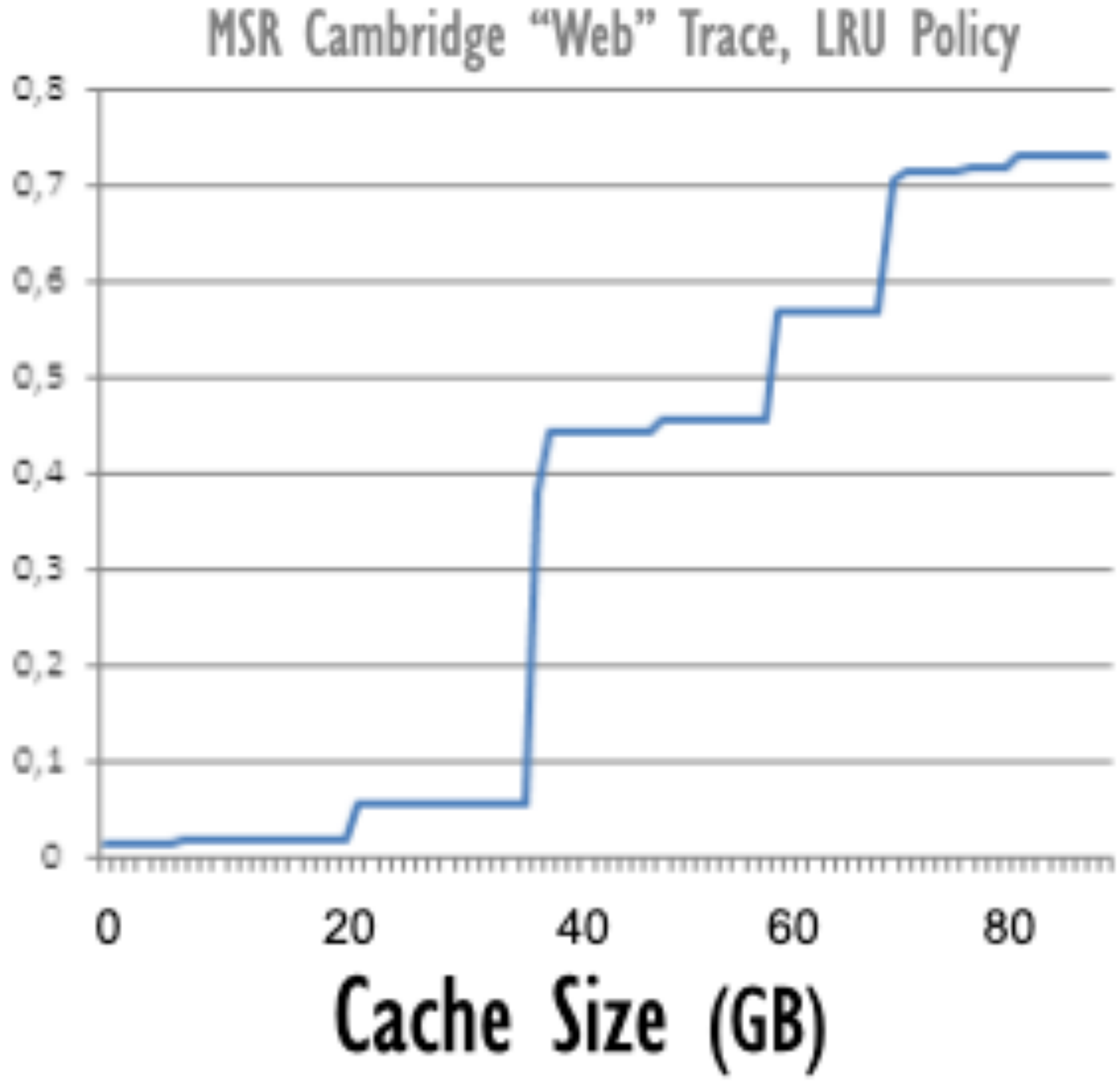
More Questions

How is my cache heuristic behaving?

- Most caches do not actually use LRU
  - e.g. Clock or ML heuristic approach



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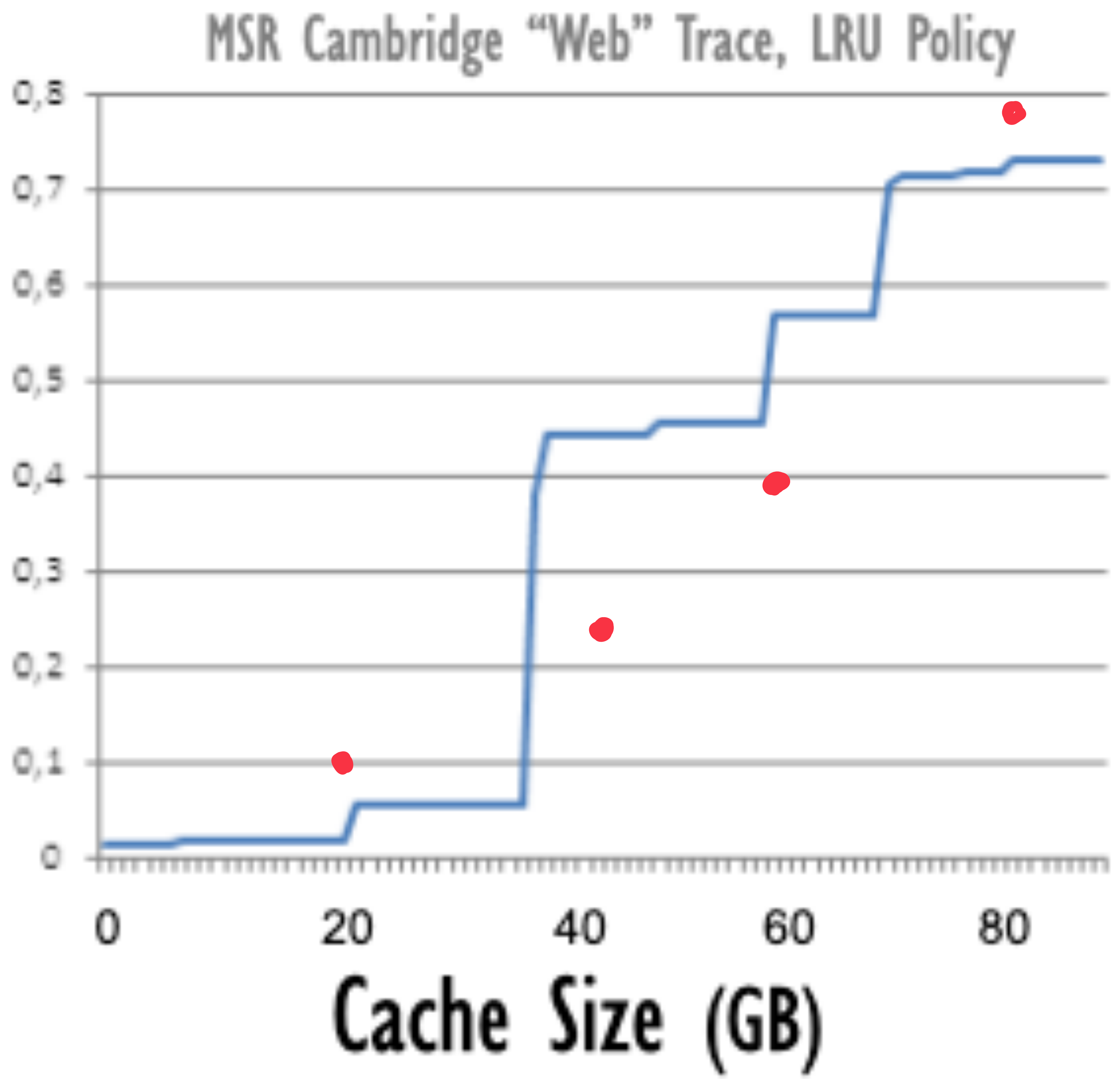

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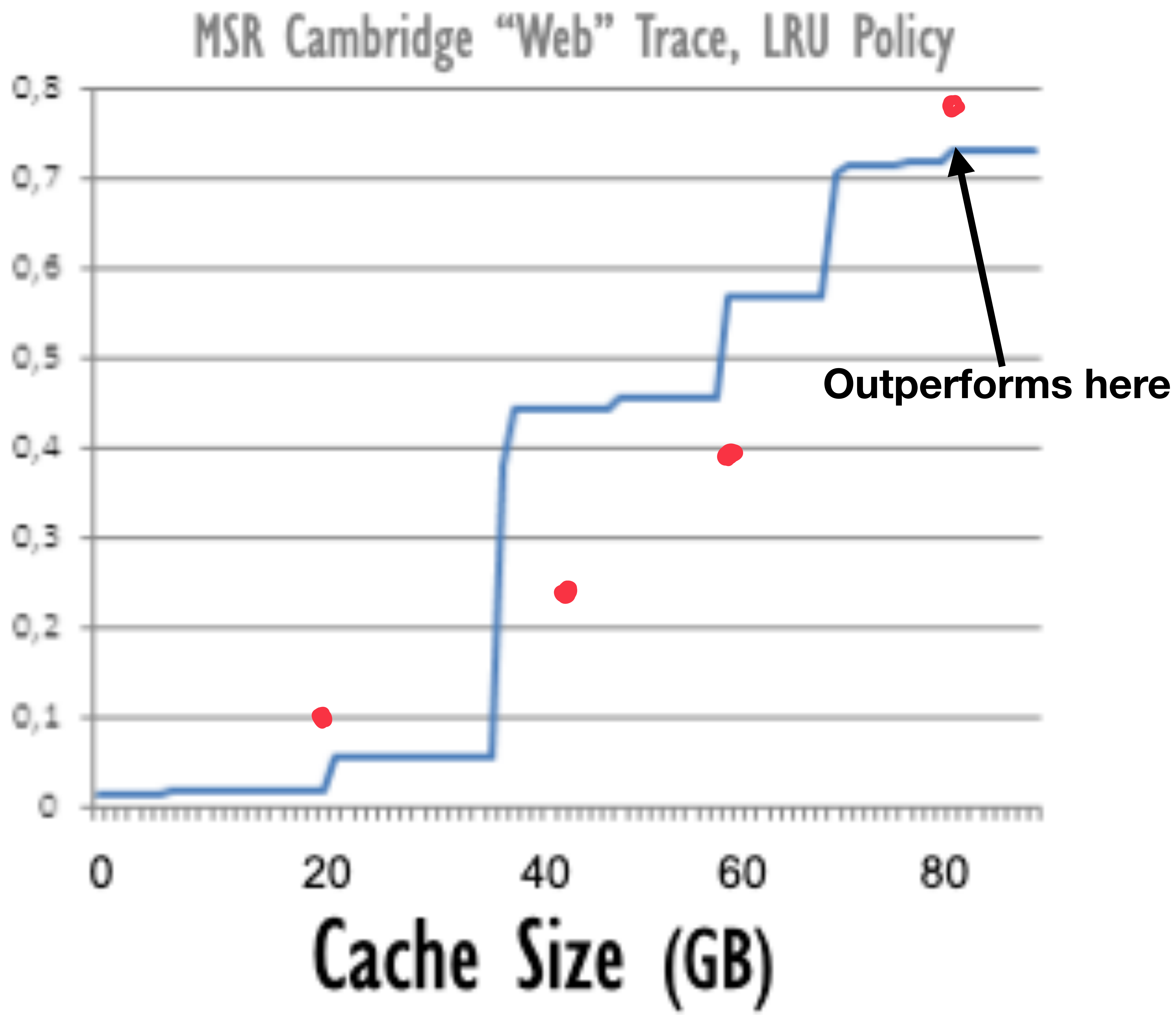

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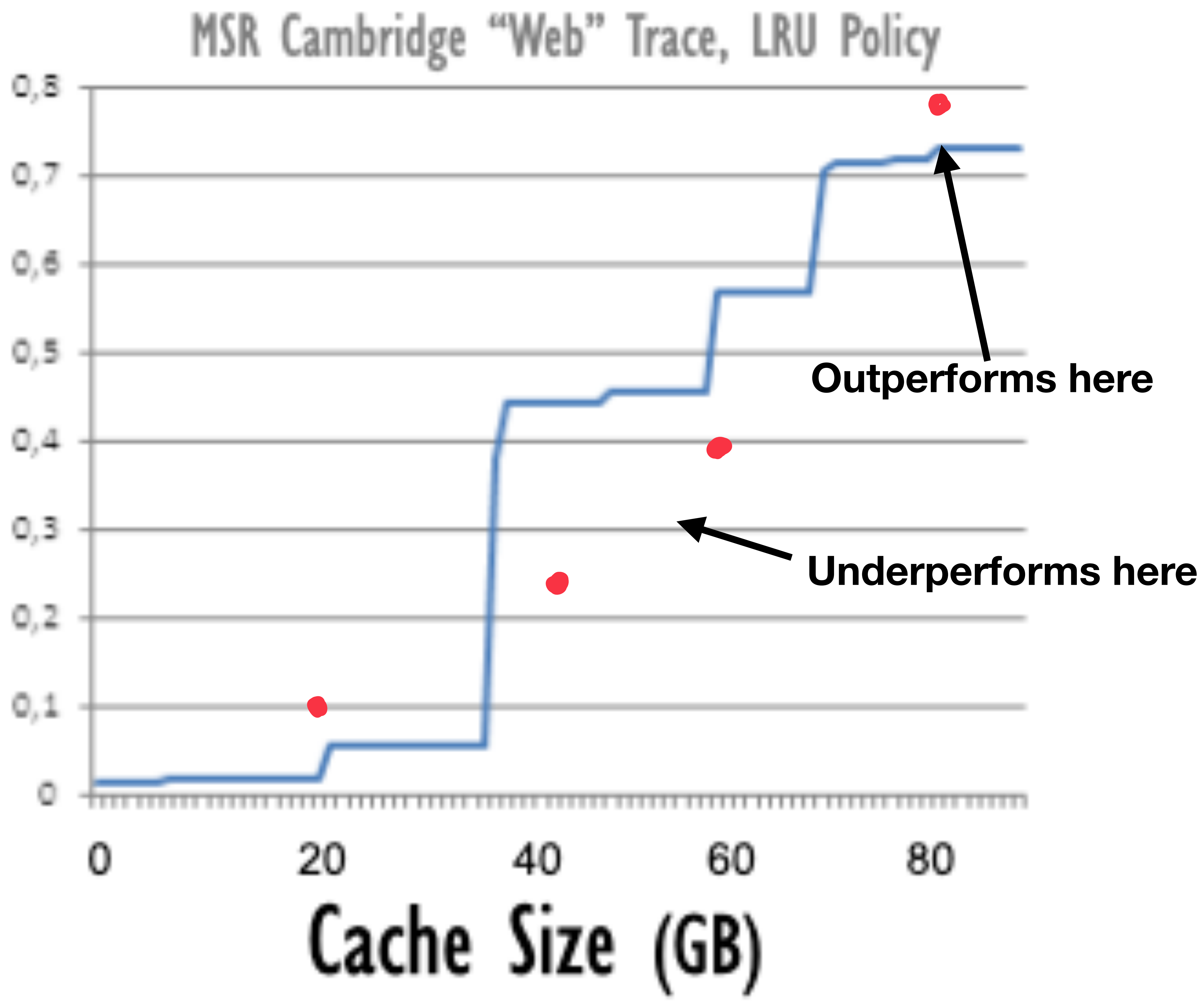
  - Or is it hurting?



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Augmented Tree Algorithms

State of the Art

- 1970 Mattson et al. compute LRU Hit-rate Curve from the stack
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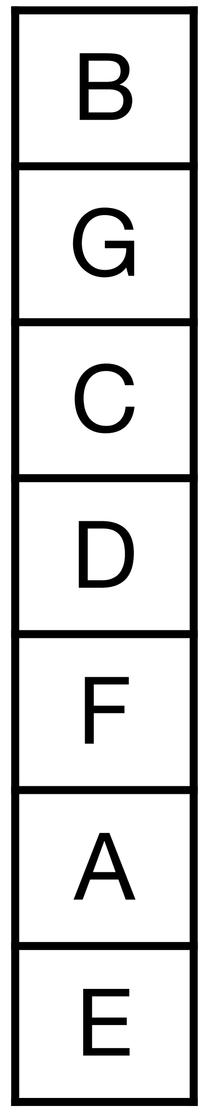
LRU Stack



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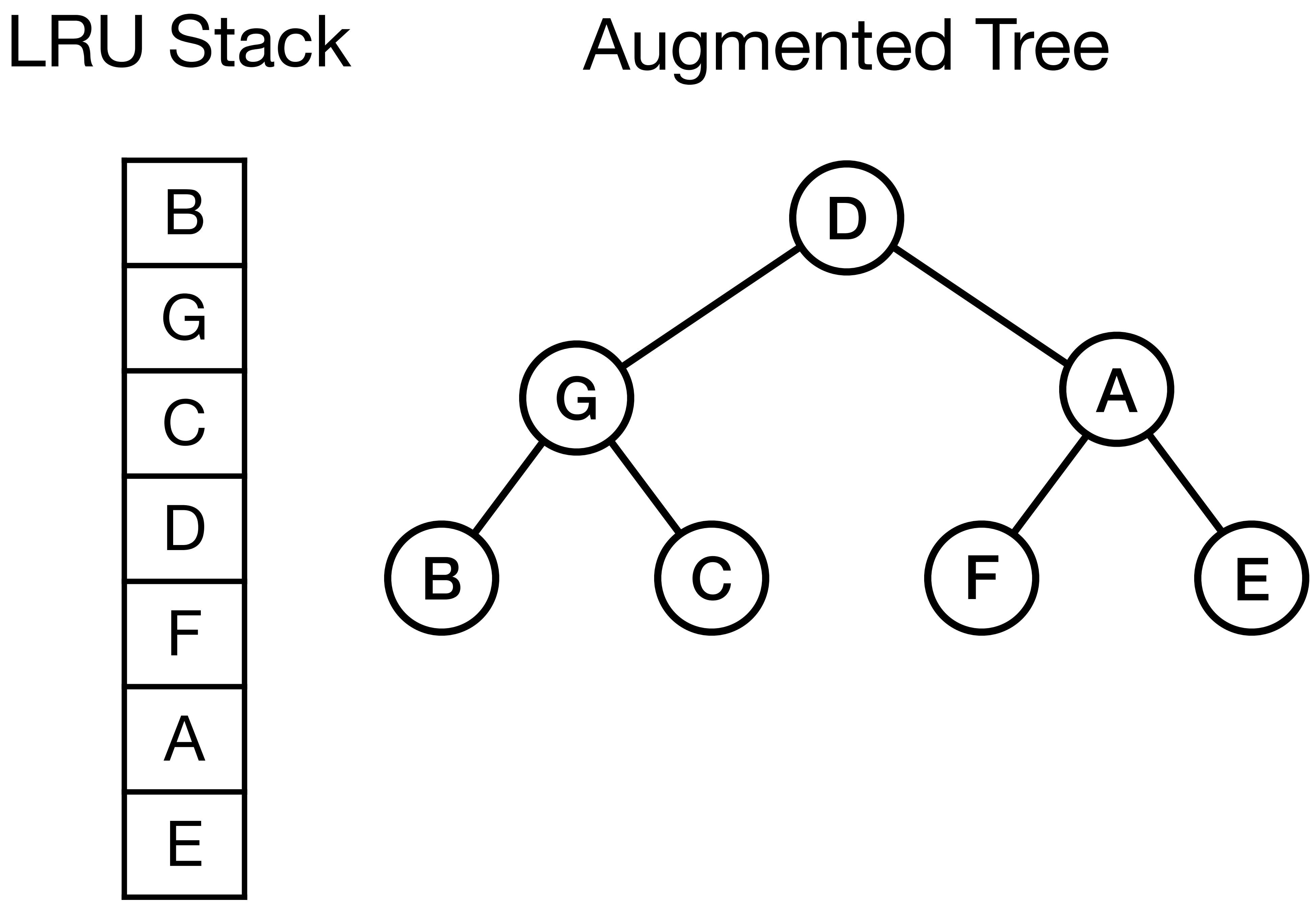
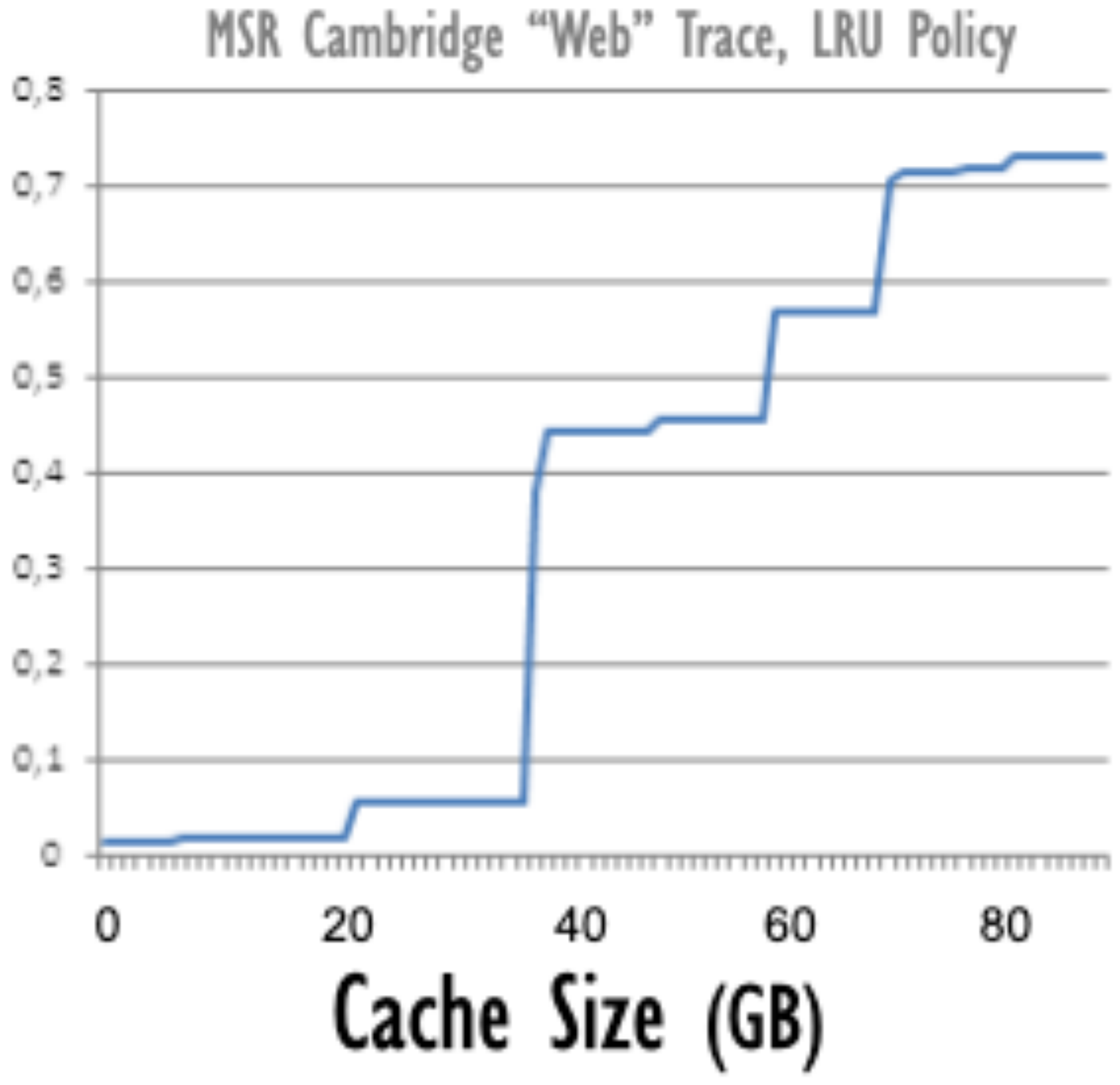
- 1975, Bennett and Kruskal store the stack as an augmented binary tree with order statistics
  - $O(n \log n)$ time algorithm
  - Best known RAM model complexity



This talk, Hit-rate Curve Computation In:

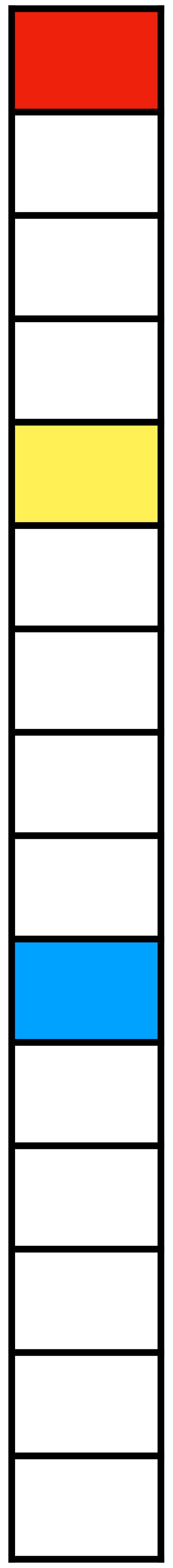
- The external-memory model
  - $\text{sort}(n) = O\left(\frac{n}{B} \log_{M/B} \frac{n}{B}\right)$ I/Os
- Parallelism
  - $O(\log^2 n)$ span
  - $O(n \log n)$ work



Lack of Locality

A Fundamental Challenge

- Accesses to the LRU stack may be random



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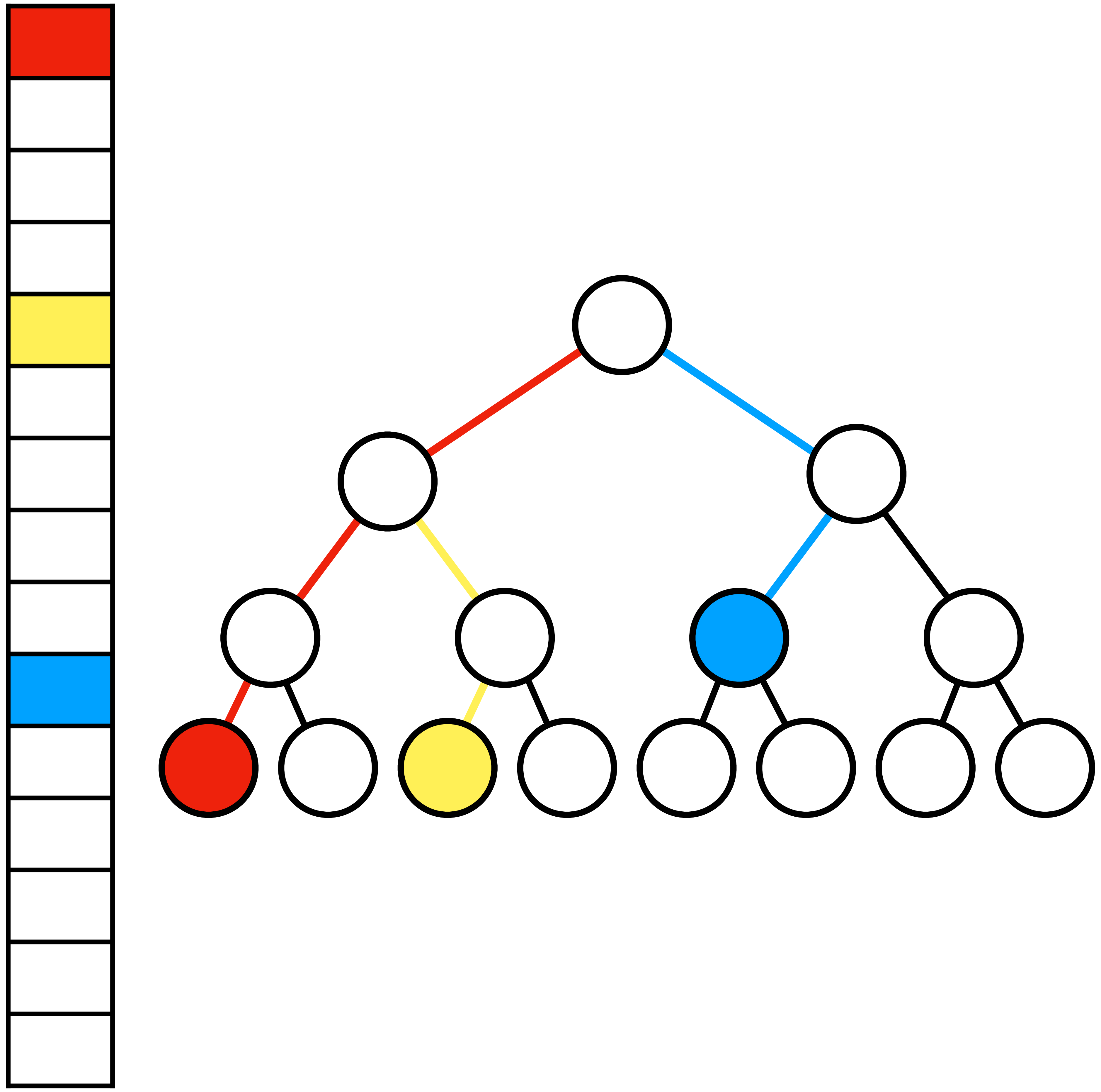

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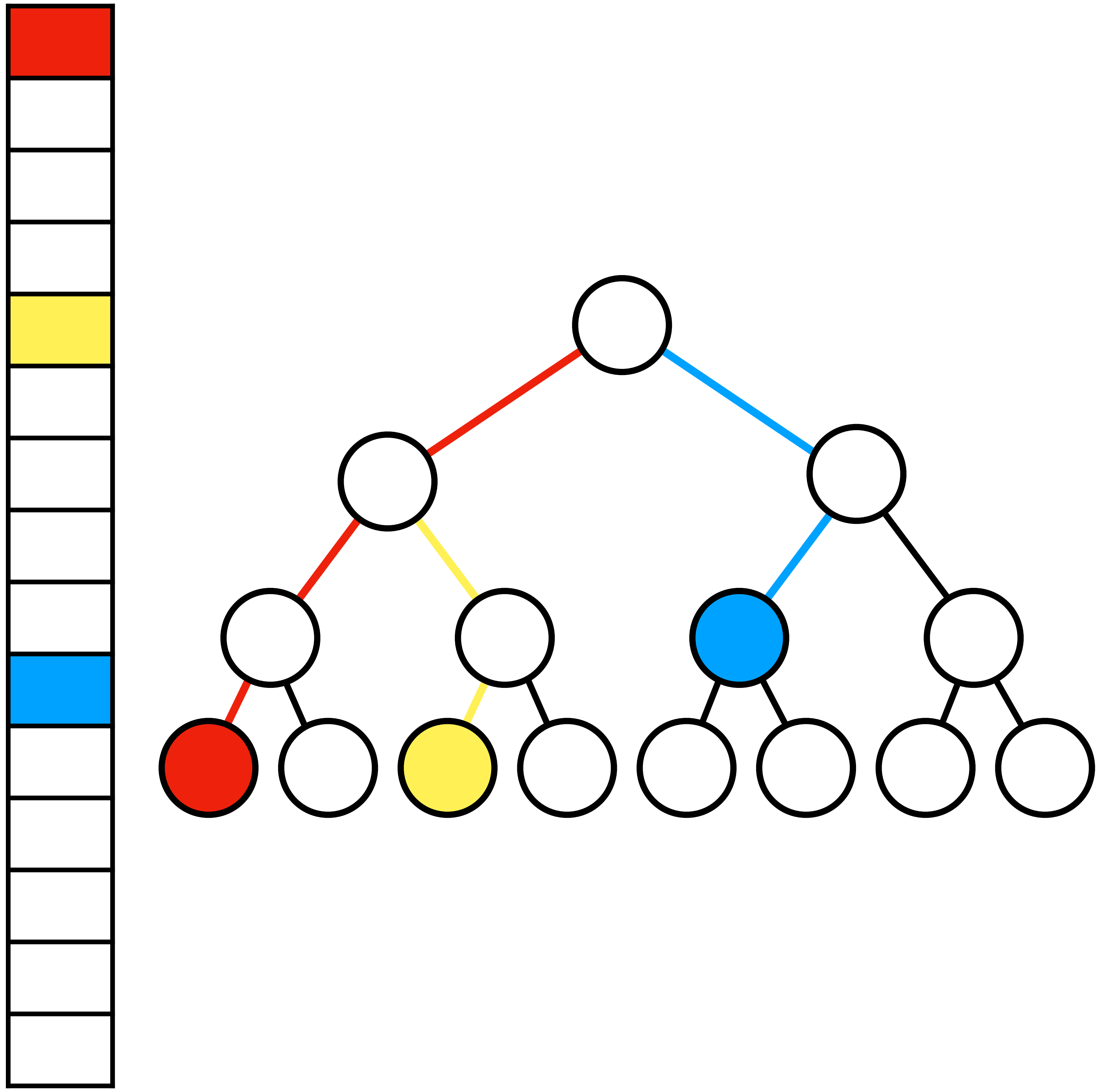
- Time to compute hit-rate curve is **100x greater** than running time of program



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PARDA: [Niu et al. 2012]

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- Perhaps not surprising, we need both parallelism and data locality

PARDA: [Niu et al. 2012]

Increment-and-Freeze

The Increment-and-Freeze Algorithm

LRU hit-rate curves with locality and parallelism

- Can surprisingly solve Hit-rate Curve without representing a LRU-stack
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- Increment-and-Freeze uses a divide-and-conquer strategy to compute the stack depth of every request

Finding Stack Distances

- Initialize an Array $A[n]$ to all zeros. Indexed by 1
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- **Stack distance**: the number of unique requests between an occurrence of a page and its next occurrence.
  - ABBBA: stack distance of first A is 2
  - ABCDA: stack distance of first A is 4

Operations

- Increment-and-Freeze consists of two operations

Operations

Surprising Stuff

- Increment-and-Freeze consists of two operations
  - Increment(i, j, r): Increment array values $[i, j)$ by r
  - Freeze(i): Freeze array value $A[i]$, prevent it from being incremented more

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- Increment-and-Freeze consists of two operations
  - Increment(i, j, r): Increment array values $[i, j)$ by r
  - Freeze(i): Freeze array value $A[i]$, prevent it from being incremented more
- Goal: After processing all operations, A contains the stack distance of each request
  - Trivial to construct hit-rate curve from stack distances

Building Operations

- Each request j becomes $I(\text{prev}(j), j, 1)$ and $F(\text{prev}(j))$

- Example: ABEBA

  0 0 0 0 0 Initialize

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3 2 2 1 0 A: I(1,5,1) F(1)

Divide and Conquer Structure

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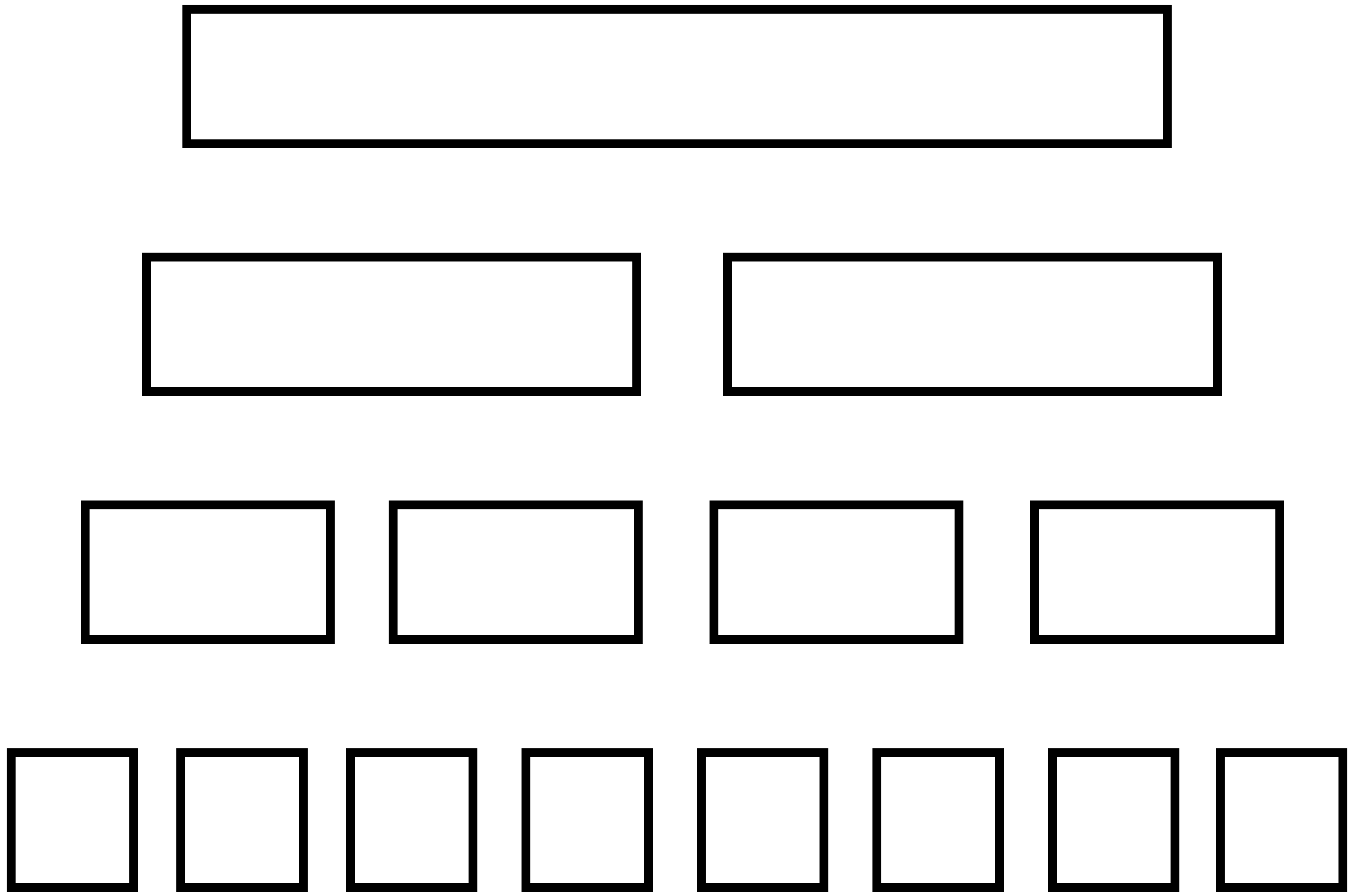
  - Can merge neighboring increments that affect the same range

- Partition procedure divides a range of request indices in half
  - Operations are restricted to only affect their respective side of the partition
  - One Increment may become two

Divide and Conquer Structure

- Divide-and-conquer performed via repeated partitions
- Even though Increments may split
    - $O(n)$ operations per level



Increment-and-Freeze Complexity

The base algorithm

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  - $O(n \log n)$ time total
- External memory model: Scan at each level, $O(\frac{n}{B} \log n)$ I/Os
- PRAM model: single-threaded partition, subproblems in other threads, thus $O(n)$ span and $O(n \log n)$ work

Lightning Round

Theoretical Extensions

See the paper :)

- External Memory: $\text{sort}(n) = O(\frac{n}{B} \log_{M/B} \frac{n}{B})$ I/Os

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- External Memory: sort$(n) = O(\frac{n}{B} \log_{M/B} \frac{n}{B})$ I/Os

- PRAM: Span $O(\log^2 n)$, work $O(n \log n)$
  - Cluster sum: cool application of parallel prefix sums

Implementation

See the paper x2 :)

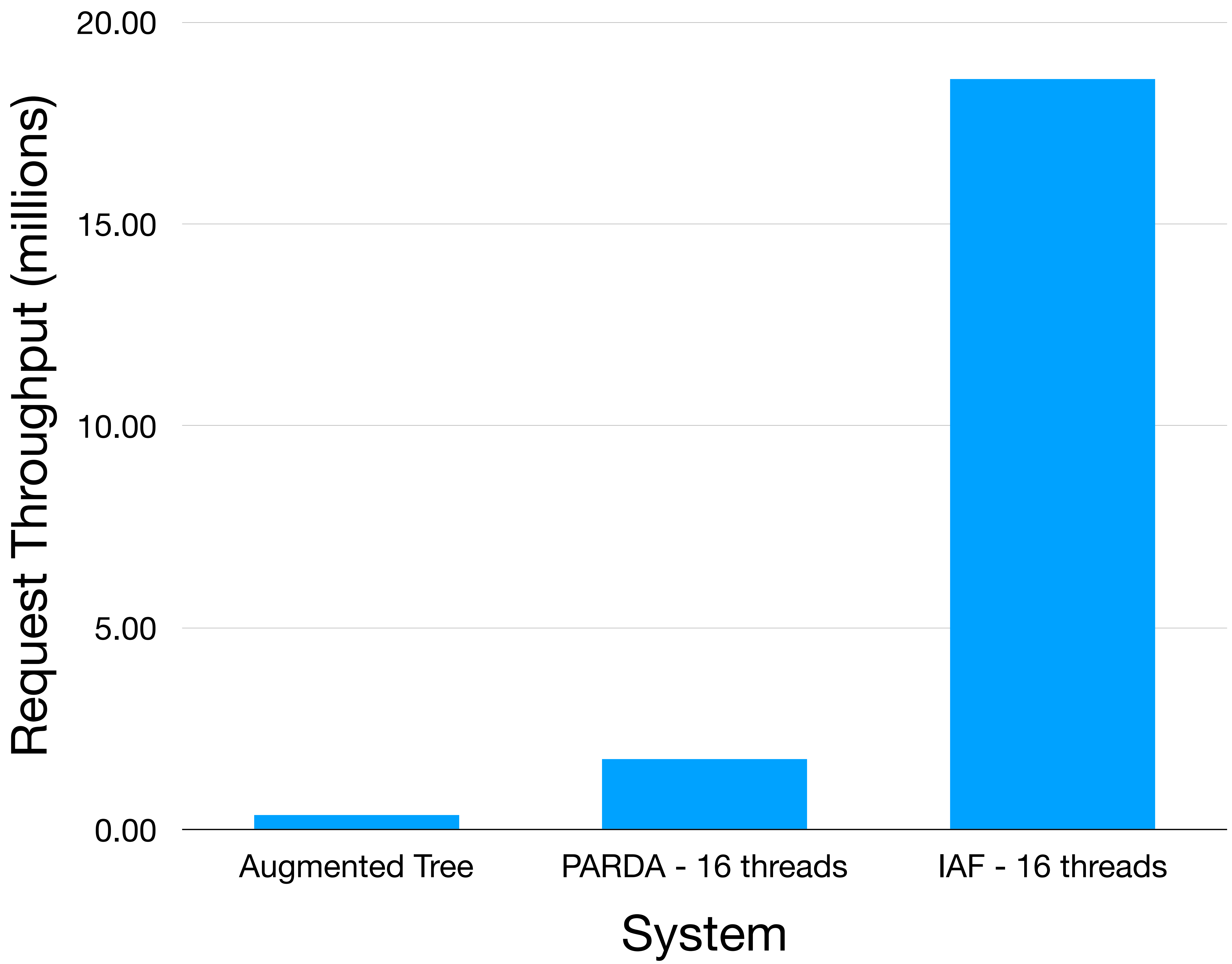
- We implemented the base Increment-and-Freeze algorithm

- Highly optimized via a number of cool tricks
  - Faster! Uses less memory!

Results

See the paper x3 :)

- Single-threaded
  - 9x faster than augmented tree
  - 8x faster than splay tree
- Cuts a 13 hour computation down to only 12 minutes



Conclusion

- Increment-and-Freeze
    - Computing LRU hit-rate curves with data locality and parallelism
- Everyone operating a cache should have real-time telemetry
    - This work has the potential to enable real-time cache analysis

More Slides

Operations

Example

- Request sequence: ABA
  - A -> $I(0,1,1)$, $F(0)$
  - B -> $I(0,2,1)$, $F(0)$
  - A -> $I(1,3,1)$, $F(1)$

- Full op sequence: $I(0,1,1)$, $F(0)$, $I(0,2,1)$, $F(0)$, $I(1,3,1)$, $F(1)$

Sampling

- Efficient approaches for computing LRU hit-rate curves down sample the key space. No quality guarantees for curve

- If we are trying to understand why our paging heuristic is underperforming, sampling may hide the answer.

- Increment-and-Freeze composes with sampling, further improving performance

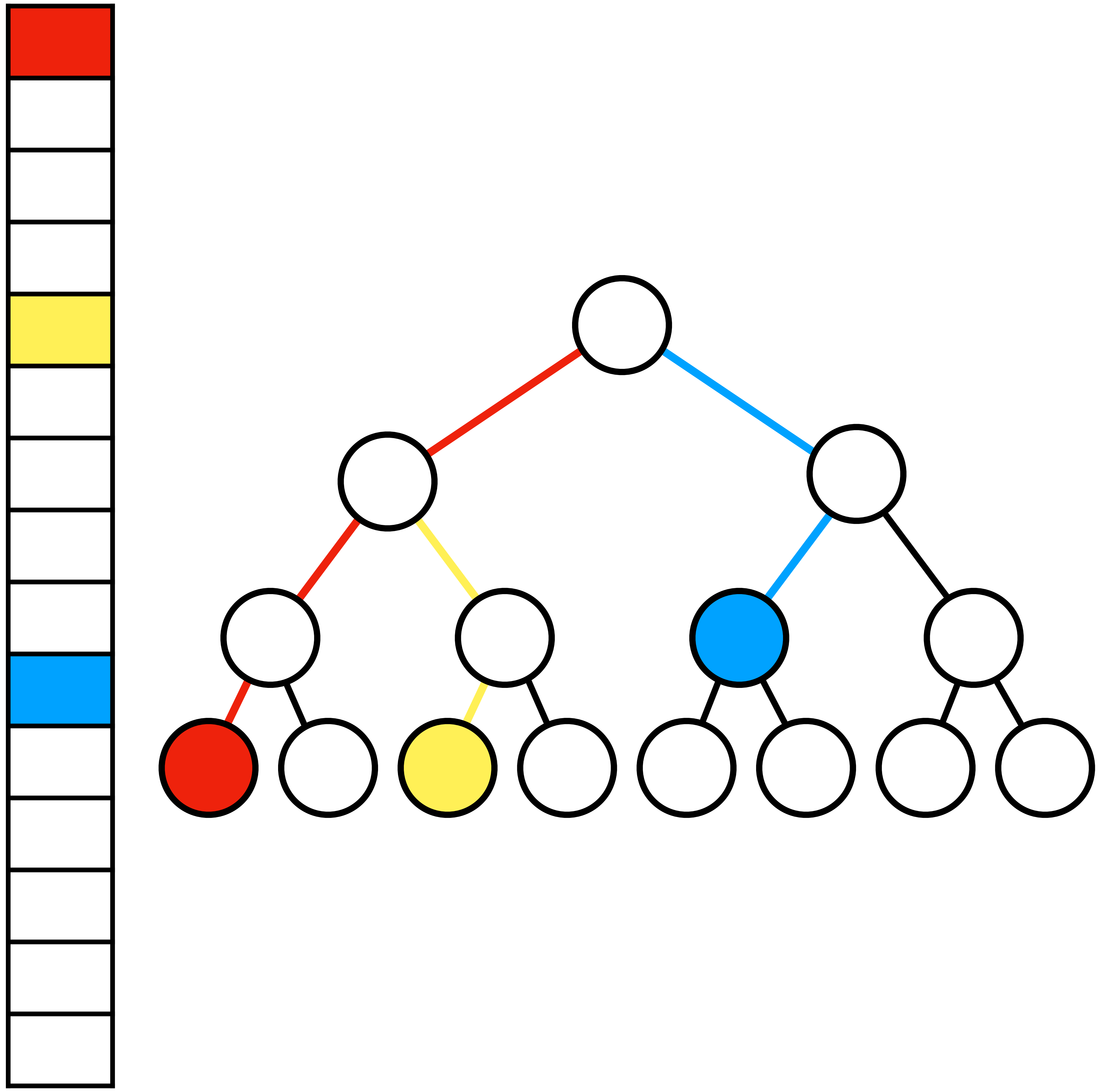
Lack of Locality

Why Hit-rate Curve Computation is 100x Slower

- Example: Building a hit rate curve for L3 cache

- At most 1 cache miss per access when running executable

- Versus $O(\log n)$ cache misses per access when producing the hit-rate curve!



Operations

Creating operations from requests

- prev(j): The index of the previous request that references the same page as j
  - For example: ABCAC, prev$(4) = 1$

Comparison with PARDA

Comparable speedup without memory cost